Reply to the Comment

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In the comment [1], Zanardi and Rasetti argue that several claims in our recent letter [2] are questionable. Here we show these claims remain true. Our points are that (i) the Hamiltonian (1) in Ref. [2] describes amplitude damping as well as phase damping; (ii) the free-Hamiltonian-elimination (FHE) has feasible implementation in practice and there exists a simple FHE procedure which is designed without the knowledge of the noise parameters $\lambda^{(i)}$; (iii) the gate operation constructed in the letter [2] is an encoded universal operation, not an encoded controlled NOT (CNOT).

(i) There are mainly two kinds of dissipation, amplitude damping and phase damping. General amplitude damping of many qubits is usually described by the following interaction Hamiltonian [3]

$$H_I = \sum_l \left(g_l^* \sigma_l^+ B_l + g_l \sigma_l^- B_l^+ \right), \tag{1}$$

where all B_l are bath operators consisting of annihilation operators. Under the rotating wave approximation, Eq. (1) is equivalent to the Hamiltonian (1) in Ref. [2] with the noise parameters $\lambda^{(1)} = Re\,(g_l)\,, \lambda^{(2)} = Im\,(g_l)$, and $\lambda^{(3)} = 0$. So the Hamiltonian there describes amplitude damping as well as phase damping. Most practical decoherence belongs to this class, though the Hamiltonian is not in the most general form. The coupling (1) in the comment can not be transformed into the Hamiltonian describing pure dephasing without changes to the free Hamiltonian of the qubits.

(ii) The FHE in Ref. [2] is designed with the assumption that we know accurately the noise parameters $\lambda^{(i)}$. These parameters are determined by the

type of the dissipation. If the noise parameters do not vary with time, they can be detected by measuring suitable observables of some test qubits subject to the same source of environmental noise. To perform the detect, we may use a technique similar to quantum tomograph of an unknown evolution of an open quantum system [4]. But FHE can also be realized without the knowledge of the noise parameters. Here we present a simple FHE technique having this advantage. We introduce a classical homogeneous far-violet-detuned optical field which acts on all the qubits. Under the adiabatic approximation, the Hamiltonian describing the driving process has the form [5]

$$H_{drv} = -\sum_{l} \frac{2\left|g\right|^2 \left|E\right|^2}{\omega_{opt} - \omega_0} \sigma_l^z, \tag{2}$$

where ω_{opt} and ω_0 are frequencies of the optical field and of the qubits, respectively. By adjusting the intensity $\left|E\right|^2$ of the optical field, we can choose the coefficient in Eq. (2) to satisfy $\frac{2|g|^2|E|^2}{\omega_{opt}-\omega_0} = \omega_0$. The free Hamiltonian of the qubits is thus eliminated. This FHE technique has another advantage, that is, it still works in the circumstance $\lambda^{(3)} = 0$, whereas the previous FHE technique fails in this case.

(iii) As was shown in [1], the parameter n_l in Eq. (10) of the letter [2] is in fact zero. However, this restriction has no influence on the construction of the universal gate operation, since the restriction is indeed satisfied there (see Eq. (16) in [2]). The operator (14) constructed in [2] is an encoded universal gate operation, not an encoded CNOT. It reduces to CNOT only when the parameters α, θ, ϕ in $V_{l_2 l_2'}(\alpha, \theta, \phi)$ (Eq. (15) in [2]) have a special value, i.e., $\alpha = \phi = 0$, and $\theta = \frac{\pi}{2}$. In contrast, for a universal gate operation, these parameters should be irrational multiples of π and of each other (see Ref. [2] and references therein).

We note after acceptance of the letter [2] that a related work [6] was done independently by Zanardi and Rasetti. They use four qubits, which are required to be decohered collectively, to encode one qubit. The scheme there deals with general dissipation, i.e., it prevents amplitude damping and phase damping at the same time. The FHE is not needed in [6], but the scheme is less efficient, and moreover, the condition of collective decoherence for four qubits is not easy to satisfy in practice [7].

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References

[1] P. Zanardi and M. Rasetti, the comment.

[2] L. M. Duan and G. C. Guo, Phys. Rev. Lett. 79, 1953 (1997).

[3] C. W. Gardiner, *Quantum Noise*, Springer-Verlag, Berlin Heidelberg (1991).

[4] J. F. Poyatos, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. 78, 390 (1997).

[5] M. J. Holland, D. F. Walls, P. Zoller, Phys. Rev. Lett. 67, 1716 (1991).

[6] P. Zanardi and M. Rasetti, LANL eprint quant-ph/9705044.

[7] L. M. Duan and G. C. Guo, LANL eprint quant-ph/9706020.